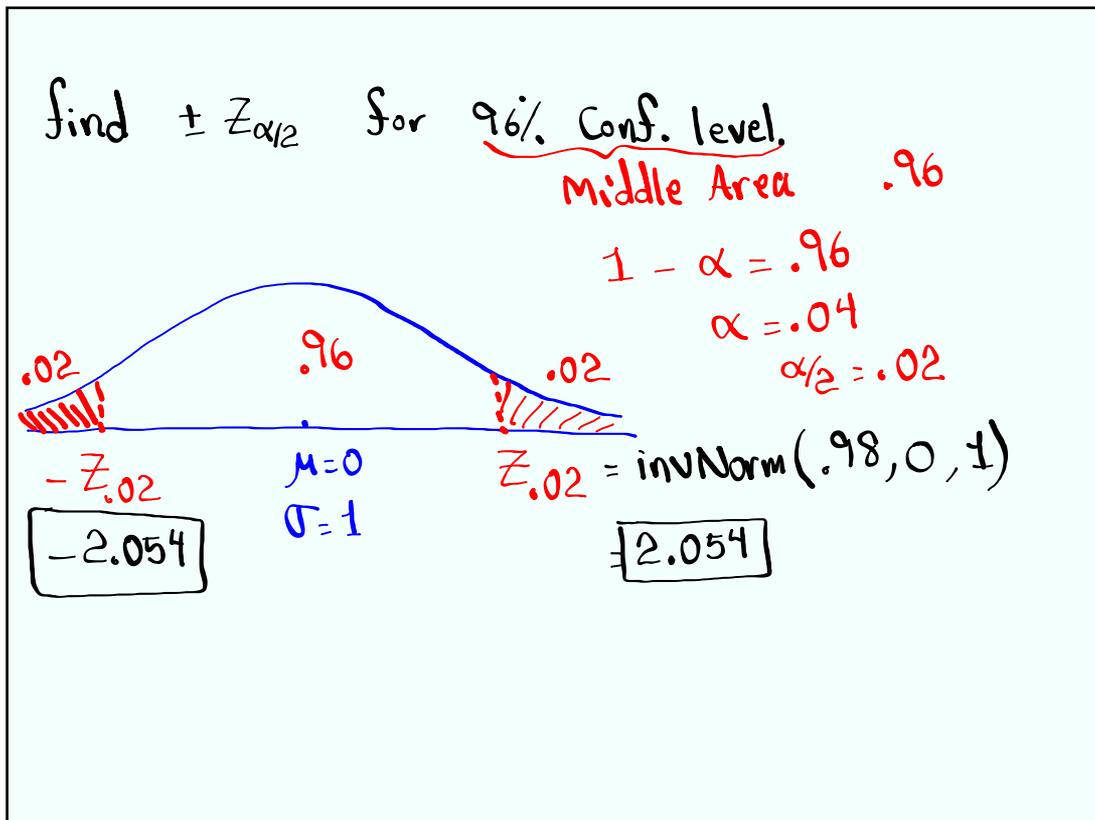


Statistics Lecture 21



Feb 19-8:47 AM



May 5-1:49 PM

In a survey of 475 drivers, 32% of them had texted while driving.

$n = 475$ $x = n\hat{p}$ $x = 475(.32) = 152$
 $\hat{p} = .32$ if decimal \Rightarrow Round-up

Find **Conf. interval** for the **proportion of all** drivers that have texted while driving.

\rightarrow NO C-level \Rightarrow use .95

1-Prop ZInt

$x = 152$ $.28 < p < .36$ we are 95% confident that between 28% and 36% of all drivers have texted while driving.

$n = 475$
 C-level = .95 $E = \frac{.36 - .28}{2} = .04$

Calculate $\hat{p} = \frac{.36 + .28}{2} = .32$

May 5-1:53 PM

how many drivers should we survey if we wish to have conf. interval @ 99% level and error not to exceed 5%?

$n = \hat{p}\hat{q} \left(\frac{Z_{\alpha/2}}{E} \right)^2 = (.32)(.68) \left(\frac{2.576}{.05} \right)^2 = 577.5779 \dots$
 $n = 578$

Suppose \hat{p} & \hat{q} were both unknown,
 use .5 for each

$n = (.5)(.5) \left(\frac{2.576}{.05} \right)^2 = 663.5776$
 $n = 664$

when working with proportion

- 1) use invNorm to find $Z_{\alpha/2}$.
- 2) use 1-Prop ZInt to find Conf. interval

May 5-2:02 PM

working with **population Mean**

$$\bar{x} - E < \mu < \bar{x} + E$$

Population Standard deviation
Sample Mean Point-estimate
Margin of error

Case I: σ Known

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

use invNorm

STAT TESTS ZInterval

inpt: Stats

May 5-2:11 PM

Given: $n=32$, $\bar{x}=84$, $\sigma=12$

Find 90% Conf. interval for Pop. mean.

$$\bar{x} - E < \mu < \bar{x} + E$$

Since σ is known

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.645 \cdot \frac{12}{\sqrt{32}}$$

$$\approx 3.5$$

STAT TESTS ZInterval

inpt: Stats

$\sigma = 12$

$\bar{x} = 84$

$n = 32$

C-level: .9

Calculate

80.5 < μ < 87.5

we are 90% confident that population mean is between 80.5 \pm 87.5

$$E = \frac{87.5 - 80.5}{2} = 3.5$$

$$\bar{x} = \frac{87.5 + 80.5}{2} = 84$$
$$Z_{.05} = \text{invNorm}(.95, 0, 1) = 1.645$$

May 5-2:15 PM

I randomly selected 28 exams and the mean score was 88. $n=28$
 $\bar{x}=88$

Find 98% Conf. interval for the mean of all exams if Standard deviation of scores of all exams is known to be 15.

C-level: .98
 $\sigma=15$

Since σ is known $\langle \mu \rangle$
 we use Z Interval

inpt: Stats
 $\sigma=15$
 $\bar{x}=88$
 $n=28$
 C-level: .98

$81 < \mu < 95$

we are 98% Confident that the mean score of all exams will be between 81 & 95.

$$E = \frac{95 - 81}{2} = 7$$

$$\bar{x} = \frac{95 + 81}{2} = 88$$

May 5-2:26 PM

working with population Mean

$$\bar{x} - E < \mu < \bar{x} + E$$

Population Standard deviation σ
 Sample Mean Point-estimate
 Margin of error

Case I: σ Known

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

use invNorm

STAT TESTS Z Interval
 inpt: Stats

Case II: σ unknown

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$\hookrightarrow df = n - 1$
 use invT

STAT TESTS T Interval
 inpt: Stats

May 5-2:11 PM

Given $n=12$, $\bar{x}=34$, $S=8$
 C-level: .98

Find conf. interval for pop. mean.

$$\bar{x} - E < \mu < \bar{x} + E$$

$$34 - 6 < \mu < 34 + 6$$

$$28 < \mu < 40$$

σ unknown
 $E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} = 2.778 \cdot \frac{8}{\sqrt{12}} \approx 6$
 $E = \frac{40 - 28}{2} = 6$
 $\bar{x} = \frac{40 + 28}{2} = 34$

$\mu=0$
 σ unknown
 $df=11$
 $t_{.01} = \text{invT}(.99, 11) = 2.778$

STAT
 TESTS
 T Interval

inpt: $\bar{x}=34$
 $S=8$
 $n=12$
 C-level: .98

28 < μ < 40

May 5-2:38 PM

I randomly selected 10 people, the mean of their credit scores was 780 with standard deviation of 35.

$n=10$, $\bar{x}=780$, $S=35$
 $df=n-1=9$
 σ unknown

Find Conf. interval for the mean credit score of all people. $755 < \mu < 805$

\rightarrow NO C-level use .95
 σ unknown \rightarrow T Interval

$$E = \frac{805 - 755}{2} = 25$$

$$\bar{x} = \frac{805 + 755}{2} = 780$$

Find $\pm t_{\alpha/2}$ for 95% C-level with $df=9$

$\mu=0$
 σ unknown
 $df=9$
 $t_{.025} = \text{invT}(.975, 9) = 2.262$

IF Your calc does not have invT command \Rightarrow Download C.Calculator App.

May 5-2:47 PM

I surveyed 15 students. Here are their ages:

28	32	18	25	40	Store in L1
20	35	19	30	48	Find \bar{x} & S.
52	38	24	18	26	Round to whole #

Find 90% Conf. interval for the mean age of all students. σ Unknown

Use $\boxed{\text{T Interval}}$

$E = \frac{35 - 25}{2} = 5$

$\bar{x} = \frac{35 + 25}{2} = 30$

$\bar{x} = 30.2 \approx 30$

$S = 10.658 \approx 11$

$n = 15 \rightarrow df = 14$

Find $\pm t_{\alpha/2}$ for 90% C-level with $df = 14$

$t_{.05} = \text{invT}(.95, 14) = \boxed{1.761}$

$\boxed{25 < \mu < 35}$

May 5-2:58 PM

How to determine minimum Sample Size needed for pop. mean:

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \Rightarrow n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

Always round-up if σ unknown \rightarrow use S.

From last example, find min. Sample Size needed if we wish 94% C-level and error not to exceed 10.

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left(\frac{1.881 \cdot 11}{10} \right)^2 = 4.28 \approx \boxed{n=5}$$

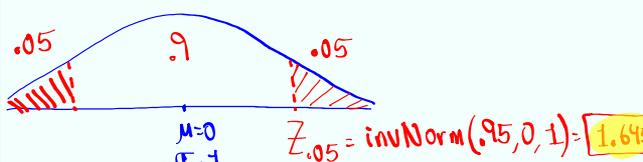
$Z_{.03} = \text{invNorm}(.97, 0, 1) = 1.881$

Redo with $E = 5$

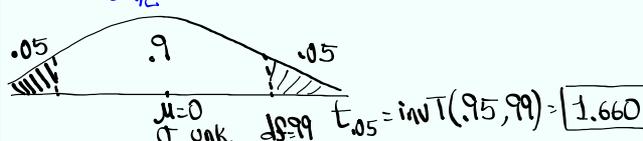
$$n = \left(\frac{1.881 \cdot 11}{5} \right)^2 \approx \boxed{18}$$

May 5-3:08 PM

Find $Z_{\alpha/2}$ for 90% C-level.



Find $t_{\alpha/2}$ for 90% C-level with $df=99$.



Find $t_{\alpha/2}$ for 90% C-level with $df=999$

Same drawing $t_{.05} = \text{invT}(.95, 999) = 1.646$

As df gets bigger & bigger
 $t_{\alpha/2} \approx Z_{\alpha/2}$

May 5-3:18 PM

I randomly selected 20 exams. Here are the scores:

80	75	100	92	98	Find $\bar{x} \approx 81$
68	55	65	72	70	
88	100	95	95	90	$S \approx 15$
82	78	93	67	50	Round to whole#

Find Conf. interval for the mean of all exams.
 C-level: .95

σ unknown \rightarrow T Interval $74 < \mu < 88$

$E = \frac{88 - 74}{2} = 7$
 $\bar{x} = \frac{88 + 74}{2} = 81$

May 5-3:25 PM

Find # of exams needed to construct
 99% Conf. interval for the mean of all
 exams and error not to exceed 5
 Points.

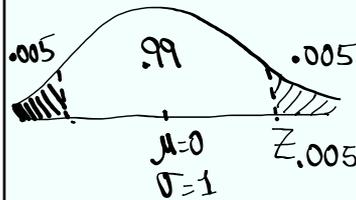
56
 21 ± .22

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

no $\sigma \rightarrow$ use S

$$n = \left(\frac{2.576 \cdot .15}{5} \right)^2$$

$$= 59.721 \dots \approx \boxed{60}$$



$$Z_{.005} = \text{invNorm}(.995, 0, 1)$$

May 5-3:31 PM